Lecture “Advanced Data Analytics”

Problem Set 6

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<u>Exercise 1</u>

<u>Gaussian Process regression with GPFLOW and SCIKIT_LEARN</u>

Look at the test functions below*. Pick **three** of those test functions.

a) Approximate a 2-dimensional function stated below with Gaussian process regression based 10, 50, 100, 500 points randomly sampled from $[0 , 1]^2$. Compute the average and maximum error. The errors should be computed by generating 1,000 uniformly distributed random test points from within the computational domain. Play with different kernels.
Use the two GP libraries we looked at in the lecture—that is, GPFlow and Scikit-learn. Do the two differ in performance when the same # of points and the same kernels are used?

b) Plot the maximum and average error as a function of the number of sample points.

c) Repeat a) and b) for 5-dimensional and 10-dimensional functions. Is there anything particular you observe?

1. OSCILLATORY: $f_1(x) = \cos\left(2\pi w_1 + \sum_{i=1}^{d} c_i x_i\right),$

2. PRODUCT PEAK: $f_2(x) = \prod_{i=1}^{d} \left(c_i^{-2} + (x_i - w_i)^2\right)^{-1},$

3. CORNER PEAK: $f_3(x) = \left(1 + \sum_{i=1}^{d} c_i x_i\right)^{-(d+1)},$

4. GAUSSIAN: $f_4(x) = \exp\left(-\sum_{i=1}^{d} c_i^2 t(x_i - w_i)^2\right),$

5. CONTINUOUS: $f_5(x) = \exp\left(-\sum_{i=1}^{d} c_i |x_i - w_i|\right),$

6. DISCONTINUOUS: $f_6(x) = \begin{cases} 0, & \text{if } x_1 > w_1 \text{ or } x_2 > w_2, \\ \exp\left(\sum_{i=1}^{d} c_i x_i\right), & \text{otherwise.} \end{cases}$

*Choose the parameters *w* and *c* in meaningful ways.

Exercise 2

Derivative modelling with GPs

Using the notebook **GP-BS-Pricing_01.ipynb**, investigate the effectiveness of a Gaussian process with RBF kernels for learning the shape of a European derivative (call) pricing function $V_t = f_t(S_t)$ where S_t is the underlying stock's spot price.

- The risk free rate is $r = 0.001$
- The strike of the call is $KC = 130$
- The volatility of the underlying is $\sigma = 0.1$
- The time to maturity $\tau = 1.0$

Your answer should plot the variance of the predictive distribution against the stock price, $S_t = s$, over a data set consisting of $n \in \{10, 50, 100, 200\}$ gridded values of the stock price $s \in \Omega^h := \{i\Delta s \mid i \in \{0, \ldots, n-1\}, \Delta s = 200/(n-1)\} \subseteq [0, 200]$ and the corresponding gridded derivative prices $V(s)$.
Each observation of the dataset, $(s_i, v_i = f_t(s_i))$ is a gridded (stock, call price) pair at time t.