Lecture "Advanced Data Analytics"

Problem Set 4

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Exercise 1

Artificial Neural Nets

You are given the feed-forward network (multi-layer perceptron) below, consisting of sigmoid-perceptrons, and the corresponding weights. Per default, x_0 = 1 for the bias.

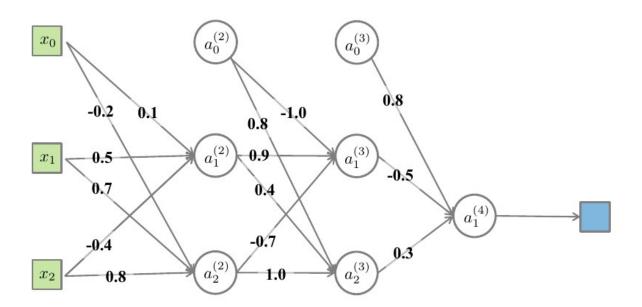
Write a piece of code that implements the network below with the weights specified.

Determine the output for the following inputs:

a)
$$x_1 = 0.7$$
, $x_2 = 1.0$

b)
$$x_1 = -0.5$$
, $x_2 = 2.5$

c)
$$x_1 = -3.0$$
, $x_2 = 3.0$



Exercise 2

Artificial Neural Nets

Consider the sigmoid activation function for a node in a neural network:

$$g(a) = \frac{1}{1 + exp(-a)}$$

Show that the derivative of this activation function takes the simple form:

$$\frac{d}{da}g(a) = g(a)\left(1 - g(a)\right)$$

Exercise 3

Artificial Neural Nets

Apply the simple multi-layer perceptron class distributed in the class (located **data/mlp.py**) to classify the MNIST data set (located **data/mnist.pkl.gz**)

To do so, proceed as follows.

- a) Describe in few words the MNIST data set.
- b) Describe every function in mlp.py with few words. What are they doing?
- c) Implement the routine below to run a test classification on the data set.

Play with different 3 sizes of test / training sets. What are the respective performances?

Exercise 4

Apply the simple multi-layer perceptron class distributed in the class (located **data/mlp.py**) to the following dataset (taken from Anscombe's quartet):

$$(x1, y1) = (10.0, 9.14), (x2, y2) = (8.0, 8.14), (x3, y3) = (13.0, 8.74), (x4, y4) = (9.0, 8.77), (x5, y5) = (11.0, 9.26), (x6, y6) = (14.0, 8.10), (x7, y7) = (6.0, 6.13), (x8, y8) = (4.0, 3.10), (x9, y9) = (12.0, 9.13), (x10, y10) = (7.0, 7.26), (x11, y11) = (5.0, 4.74).$$

- a) Use the neural network library **mlp.py** to show that a feedforward network with one hidden layer consisting of one unit and a feedforward network with no hidden layers, each using only linear activation functions, do not outperform linear regression based on ordinary least squares (OLS; for OLS, you may use the scikit library).
- b) Also demonstrate that a neural network with a hidden layer of three neurons using the tanh activation function and an output layer using the linear activation function captures the non-linearity and outperforms the linear regression.